Vol. 3, No. 1, pp. 1-15, (June 2025)
DOI: 10.21608/astb.2024.335620.1012
ASWAN SCIENCE AND TECHNOLOGY BULLETIN (ASTB)
Online ISSN: 3009-7916, Print ISSN: 1110-0184
Journal homepage: https://astb.journals.ekb.eg/ E-mail: essamshaalan@sci.aswu.edu.eg
Original Article

Multi-Commodity Flow Network Reliability Evaluation with Multiple Constraints

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Received: 11/11/2024 Revised: 28/11/2024 Accepted: 02/12/2024

Abstract

Capacitated stochastic flow networks have applications in telecommunications, transportation, and supply chain management. They are designed to manage uncertain demand and supply, making them suitable for volatile environments. Evaluating their reliability involves understanding probability theory and network dynamics, assessing the probability of successful transmission from source to destination. Advanced mathematical models and simulations are used to estimate performance and reliability. The primary purpose of this paper is to evaluate multi-commodity flow network reliability with multiple constraints. I.e. Calculate the probability that the required amount of multiple commodities can be transported simultaneously through a stochastic flow network under budget and tolerable error rate constraints. Based on minimal paths the proposed formulation is used to identify all lower boundary points necessary for the requirements. Subsequently, system reliability can be computed using these lower boundary points. To show its validity and efficiency, the proposed formulation has been used to evaluate the reliability of multi-commodity four-node network with six arcs under error rate constraint. Then, evaluate the reliability of multi-commodity four-node, five-node, and six-node networks with budget and error rate constraints.

Keywords: Flow Networks, Multi-commodity, Reliability Evaluation, Multiple Constraints.

1. Introduction

Reliability evaluation refers to the process of assessing how dependable and consistent a system, component, or process is over time.

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This involves determining the likelihood that the system will perform its intended function without failure under specified conditions for a designated period. Reliability evaluation often includes methods such as statistical analysis, testing, and modeling to predict and improve performance.

It is crucial in various fields, including engineering, manufacturing, and software development, to ensure safety, efficiency, and customer satisfaction. Evaluating reliability by employing advanced statistical models and methodologies based on simulating random variables, as Monte Carlo simulations (Qiujing and Daniel,2017; Zhifu and Xiaoping,2016; Cheng 2024). These methods provide a comprehensive understanding of potential system behaviors and help identifying the likelihood of different outcomes. Additionally, reliability can be optimized by implementing fault tree analysis (FTA) (J.F.W.Peeters, et al.,2018 and Yang, et al.,2014) and failure mode and effects analysis (FMEA)(Kapil and Shobhit,2018 and Hafidh, et al.,2024). These systematic approaches help in identifying potential failure points within a system and evaluating their consequences. By prioritizing these risks, organizations can allocate resources more effectively to mitigate them, leading to improved reliability.

The Reliability evaluation of a capacitated-flow network assesses its performance and dependability, focusing on the likelihood of meeting desired performance under varying conditions. It analyzes factors like link failure probabilities and their impact on overall performance, as well as the network's ability to reroute flow. This evaluation uses statistical methods and simulations to predict reliability, essential for designing robust networks and planning maintenance to minimize disruptions. Lin, et al., (1995) and Majid (2023) show reliability evaluation. Lin (2001) and Risat, et al., (2024) extended the stochastic-flow concept to include failures in both arcs and nodes, introducing an algorithm to find lower boundary points for d and calculate system reliability. This approach can be applied to real-world systems like telecommunications and logistics.

In the context of reliability evaluation, budget constraints refer to the limitations on financial resources that can be allocated to ensure the reliability of a system or network. These constraints impact how much can be invested in various reliability-enhancing measures, such as redundancy, maintenance, testing, and upgrades.

The system reliability of a flow network is defined when considering the transmission cost as the probability that a single commodity *(d)* will be transmitted from the source node to the sink node so that the total transmission cost is less than or equal to *(C)*. This can be computed in terms of the minimal path vectors to level *(d, C)* (called *(d, C)*-MPs. (Lin 1998; Cheng, et al., 2023; Lin 2004; Xia 2023) have extended the concept of the quickest way to determine the reliability of a stochastic flow network.

The file is considered correctly transmitted if the received version matches the original. In fact, data transfer is done through packet transmission. In reliability evaluation, the Transmission Error Rate (TER) refers to the frequency of errors that occur during the transmission of data over a communication channel. It is a critical metric for assessing the reliability and

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performance of data transmission systems, such as networks, telecommunications, and digital communications. A high transmission error rate indicates poor reliability of the communication system, leading to data corruption, loss of information, and the need for retransmissions, which can degrade overall system performance. Lin and Huang, (2013) evaluated network reliability of an SFN as the probability that a specified amount *(d)* of flow can be successfully transmitted from source to target without the error rate of received data exceeding the tolerable error rate *(E)*. An algorithm utilizing minimal paths (MPs) is suggested to identify all lower boundary points for *(d, E)*.

Elgamal, et al., (2023) evaluate network reliability as the probability that a specified flow can be transmitted from source to target at a cost *(C)*, while keeping the error rate within a tolerable limit *(E)*.

Numerous studies have evaluated network reliability using minimal cuts (PC Chang 2022; Jane, et al., 1993; Lin 2002; Yan and Qian, 2007; Lin 2010; Lin and Yeh, 2011) or minimal paths (MPs) (Xinxin, et al., 2023; Lin, et al., 1995; Lin 2001; Yeh 2005; Yeh 2002; Lin 2004; Lin 2007). An MP connects the source to the target without cycles and is used to determine flow assignments in a network.

The two-commodity reliability evaluation for a stochastic-flow network with node failure involves assessing the probability that a network can meet the demands for two commodities despite random node failures. In such networks, node and edge capacities vary according to probability distributions, reflecting real-world uncertainties. Node failures add complexity by disrupting overall network performance. The purpose of paper (Lin 2002 and Noha, et al., 2020) is to extend the reliability problem to a two-commodity case for a stochastic-flow network with node failure and how the system reliability can be calculated. The demands of commodity 1 and 2 at sink t are d^1 and d^2, respectively.

The minimum cost flow (MCF) problem determines the least cost shipment of commodities through a flow network, satisfying demands from available supplies. When generalized to multiple commodity types, it's known as the multi-commodity minimum cost flow (MMCF) problem. Lin (2001) and Suchi and Suyel (2021) extend previous reliability models to a multi-commodity case with budget constraints, focusing on a two-commodity case. They have extended it to a multi-commodity reliability model with budget constraint. The suggested approach can be summarized as follows. By giving the demand (d^1, d^2) at sink t and the budget C, an algorithm has been proposed to identify all the lower boundary points for (d^1, d^2, C) in terms of minimal paths (MPs). An MP is a path whose proper subsets can't be paths. The system reliability which is denoted by $R_{(d^1, d^2, C)}$ can then be calculated in terms of such lower boundary points for (d^1, d^2, C).

In this paper, we extend a multi-commodity reliability model with budget constraint and a transmission error rate. The purpose of this paper is to evaluate network reliability of an SFN as the probability that a specified amounts d^1 and d^2 of flow can be successfully transmitted from source to target under budget constraint c and a transmission error rate E. An algorithm

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based on MPs is proposed to find all lower boundary points for (d^1, d^2, C, E), each of which is a minimal capacity vector such that the network delivers demands d1 and d2 under the budget constraint c and the tolerable error rate E. Network reliability can then be computed by the recursive sum of disjoint products (RSDP) algorithm using all lower boundary points for (d^1, d^2, C, E).

The rest of this paper is structured as follows. *Acronyms, Notations, and Assumptions* are given in Section 2. Section 3 explains the problem formulation. Capacity vector and reliability calculations are presented in Section 4 and 5 respectively. Section 6 presents Results and discussion. Finally, conclusions are presented in section 7.

2. Acronyms, Notations, and Assumptions

2.1. Acronyms

mp Minimal path
RSDP Recursive sum of disjoint products
SFN Stochastic flow network
TER Transmission error rate
MCF Minimum cost flow
MMCF Multi-commodity minimum cost flow

2.2. Notations

s Source node
t Target node
a_i. The component (arc or node) i.
X Capacity vector, $X = (x_1, x_2, \ldots, x_n)$.
mp_j Minimal path no. j, $j= 1, 2, \ldots, m$.
w_i^j The wight of commodity j on component a_i.
M_i The maximum capacity of a component a_i.
d^j Required demand of commodity j.
F^j Flow vector, $F^j = (f_1^j, f_2^j, \ldots, f_m^j)$, where f_m^j is the current flow of commodity j on path m.
C Budget amount.
E Acceptable error rate.
e_i Transmission error rate on component a_i.

2.3. Assumptions

(i) The commodities from s to t have different types.

(ii) Each state k *of a* components a_i has a capacity c_k and a probability p_k.

(iii) flow conservation law, (Ford and Fulkerson, 1963), is satisfied for each commodity type.

(iv) The capacities of components are statistical-independent.

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3. Problem formulation

The following context illustrates the main idea to evaluate reliability of a multi-commodity flow network in terms of minimal paths under budget and tolerable error rate constraints. The basic formulation of generating flow vectors discussed in (Lin 2002). While considering tolerable error rate in generating flow vectors are studied by (Lin and Huang, 2013) to evaluate reliability of a single commodity flow network.

In this study, we will generalize the formulation in (Lin and Huang, 2013) to be used in generating flow vectors in the two-commodity case $(d^j, j = 1,2)$. In addition, our formulation considering both budget and error constraints. The proposed formulation is as follows:

$$\sum_{j=1}^{m} f_j^1 = d^1 \quad \text{and} \quad \sum_{j=1}^{m} f_j^2 = d^2 \tag{1}$$

$$\left[\sum_{a_i \in mp_j} (w_i^1 \cdot f_j^1 + w_i^2 \cdot f_j^2)\right] \leq M_i \; for\ each\ i = 1, 2, \dots, n \tag{2}$$

$$\sum_{i=1}^{n} \{c_i^1 \cdot \sum_{a_i \in mp_j} f_j^1 \; + \; c_i^2 \cdot \sum_{a_i \in mp_j} f_j^2\} \leq C \tag{3}$$

$$\frac{\sum_{j=1}^{m} ((f_j^1 + f_j^2) \prod_{a_i \in p_j} (1 - e_i))}{\sum_{j=1}^{m} f_j^1 + \sum_{j=1}^{m} f_j^2} \leq (1 - E) \tag{4}$$

The equations (1)-(2) are used to generate feasible solutions of the flow vectors for two-commodity to evaluate R_{d^1,d^2}, (Lin 2002). Flow vectors for two-commodity under budget constraint can be deduced using (1)-(3) to evaluate $R_{d^1,d^2,C}$, (Lin 2001). The purpose of this study is to evaluate $R_{d^1,d^2,E}$ and $R_{d^1,d^2,C,E}$ using the proposed formulations. I.e. use equations (1-2) and (4) to evaluate $R_{d^1,d^2,E}$ while using equations (1-4) to evaluate $R_{d^1,d^2,C,E}$.

4. Capacity vector

Let X=(x_1, x_2, …, x_n) be the capacity vector corresponding to the feasible solution (***F₁*, *F₂***), where x_i is the (current) capacity of component a_i, is given by:

$$x_i = \left[\sum_{a_i \in mp_j} (w_i^1 \cdot f_j^1 + w_i^2 \cdot f_j^2)\right] \quad for\ each\ \ i = 1, 2, \dots, n \tag{5}$$

5. System Reliability Evaluation

Let all lower boundary points $X^1, X^2, \dots, X^l$ generated by solving the above formulation after removed the non-minimal ones if the network is cyclic, (Lin, et al., 1995 and Lin 2001), then the system reliability $R_{D,C,E}$ is calculated by equation (8) :

$$R_{D,C,E} = pr \bigcup_{i=1}^{l} \{Z | Z \; \geq \; X^i\} \tag{6}$$

Where $D = (d^1, d^2)$ and $pr\{Z\} = pr\{z_1\}.pr\{z_2\}.\ldots.pr\{z_{neq}\}$. Then, we use the recursive sum of disjoint products (RSDP) procedure presented in (Chang 2019; Zuo, et al., 2007; Bai, et al., 2015; Esha and Neeraj,2023), in addition can be solved using inclusion-exclusion (Lin, et al., 1995; Lin 2001; Janan 1985; Wei 2023; Ali and Motaz, 2023). In addition, the relation given in (8) can be used to evaluate $R_{D,E}$.

6. Results and discussion

The following subsections describe how to apply the proposed formulation to evaluate both $R_{d^1,d^2,E}$ and $R_{d^1,d^2,C,E}$.

6.1. Evaluate $R_{d^1,d^2,E}$

The results of applying the error constraint on the four-node network with six arcs given in Table 2. for the network of Fig. 1. with considering node's failure, The arcs are numbered in the order of a_1 to a_6 and the nodes in the order of a_7 to a_{10}. The component data and Transmission error rate are listed in Table 1. Set w_i^1=1 and w_i^2=1.5 for each i. There are four MPs: $mp_1 = \{a_7, a_1, a_8, a_2, a_{10}\}$, $mp_2 = \{a_7, a_1, a_8, a_3, a_9, a_6, a_{10}\}$, $mp_3 = \{a_7, a_5, a_9, a_6, a_{10}\}$ *and* $mp_4 = \{a_7, a_5, a_9, a_4, a_8, a_2, a_{10}\}$. The demands are (d^1, d^2)=(3,2) , then (d^1, d^2)=(2, 2) and last case(d^1, d^2)=(2, 3).

Table 1. The components of Fig. 1 network

a_i	e_i	Capacity	Probability	a_i	e_i	Capacity	Probability
		0	0.01			0	0.005
		1	0.04			1	0.005
a_1	0.005	2	0.05			2	0.005
		3	0.9	a_7	0.004	3	0.005
						4	0.01
		0	0.01			5	0.02
		1	0.01			6	0.04
a_2	0.003	2	0.02			7	0.91
		3	0.04				
		4	0.92			0	0.01
						1	0.01
		0	0.01	a_8	0.005	2	0.02
a_3	0.006	1	0.04			3	0.04
		2	0.05			4	0.92
		3	0.9				
						0	0.01
		0	0.01			1	0.01
		1	0.01	a_9	0.008	2	0.01
a_4	0.002	2	0.01			3	0.05
		3	0.05			4	0.92
		4	0.92				
						0	0.005
		0	0.01			1	0.005
		1	0.01			2	0.005
a_5	0.008	2	0.02	a_{10}	0.006	3	0.005
		3	0.04			4	0.01
		4	0.92			5	0.02
						6	0.04
		0	0.01			7	0.91
		1	0.01				

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a_6	0.004	2	0.01
		3	0.05
		4	0.92

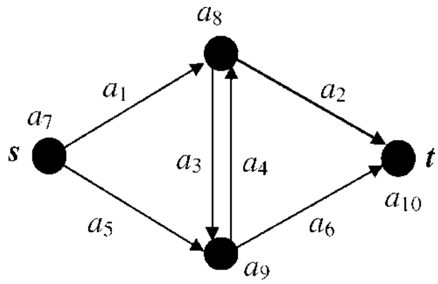

Fig. 1. A simple network

Table 2. The results of Fig. 1 network

E	$R_{(d^1,d^2,E)}$		
	(d^1,d^2) = (3,2)	(d^1,d^2) = (2, 2)	(d^1,d^2) = (2, 3)
1*	0.7430	0.8573	0.5613
0.1	0.7430	0.8573	0.5613
0.2	0.7430	0.8573	0.5613
0.3	0.7430	0.8573	0.5613
0.4	0.7430	0.8573	0.5613
0.5	0.7430	0.8573	0.5613
0.6	0.7430	0.8573	0.5613
0.7	0.7430	0.8573	0.5613
0.8	0.7430	0.8573	0.5613
0.9	0.7430	0.8573	0.5613
0.01	0	0	0
0.02	0	0	0
0.03	0.7430	0.8564**	0.5613
0.04	0.7430	0.8573	0.5613
0.05	0.7430	0.8573	0.5613
0.06	0.7430	0.8573	0.5613
0.07	0.7430	0.8573	0.5613
0.08	0.7430	0.8573	0.5613
0.09	0.7430	0.8573	0.5613

*no error considered

**Less than reliability value with no error constraint

6.2. Evaluate $R_{d^1,d^2,C,E}$

6.2.1. Four-node example

The results of four-node network. At the network of Fig. 2. The capacity, probability and transmission error rate distribution of each arc are shown in Table 3. In this example, we have four MPs with $mp_1=\{a_1, a_2\}$, $mp_2=\{a_1, a_3, a_6\}$, $mp_3=$ {a5, a4, a2} and $mp_4=\{a_5, a_6\}$. The demand is (d^1, d^2)=(2,1) and The budget is K=180 US dollars. The results are listed in Table 4.

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Table 3. The Arc Data of four-node network

a_i	e_i	Capacity	Probability	w_i^1	w_i^2	c_i^1	c_i^2
		0*	0.01				
a_1	0.05	1	0.04	1	2	20	30
		2	0.95				
		0	0.01				
a_2	0.03	1	0.03	1	1.5	30	40
		2	0.04				
		3	0.92				
		0	0.01				
a_3	0.06	1	0.04	1	1	30	40
		2	0.95				
		0	0.01				
a_4	0.02	1	0.03	1	1.5	20	40
		2	0.06				
		3	0.9				
		0	0.01				
a_5	0.08	1	0.03	1	1.5	10	20
		2	0.06				
		3	0.9				
		0	0.01				
		1	0.04				
a_6	0.04	2	0.05	1	2	20	30
		3	0.9				

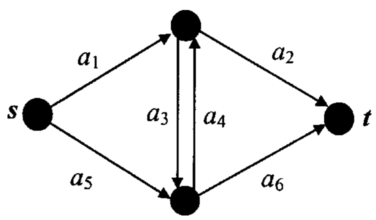

Fig. 2 A bridge network

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Table 4. The results of Fig. 2 network

E	$R_{(d^1,d^2,C,E)}$	
	(d^1,d^2) = (2,1)	(d^1,d^2) = (3, 1)
1*	0.9246	0.7387
0.1	0.9246	0.7387
0.2	0.8317	0.7387
0.3	0.9246	0.7387
0.4	0.9246	0.7387
0.5	0.9246	0.7387
0.6	0.9246	0.7387
0.7	0.9246	0.7387
0.8	0.9246	0.7387
0.9	0.9246	0.7387

6.2.2. Five-node example

The results of five-node network. The network and arc data are listed in Fig. 3 and Table 5, respectively. with transmission budget, K=800. There are six minimal paths: $mp_1=\{a_1,a_2\}$, $mp_2=\{a_4, a_5\}$, $mp_3=\{a_4, a_3, a_2\}$, $mp4=\{a_6,a_8\}$, $mp_5=\{a_6, a_7, a_5\}$ and $mp_6=\{a_6, a_7, a_3, a_2\}$. If the demand is set to be (d^1, d^2) and we have different demands. The results are listed in Table 6.

Table 5. The Arc Data of five-node network

a_i	e_i	Capacity	Probability	w_i^1	w_i^2	c_i^1	c_i^2
		0	0.01				
		1	0.02				
a_1	0.05	2	0.02	1	1.5	30	40
		3	0.02				
		4	0.03				
		5	0.9				
		0	0.01				
		1	0.02				
a_2	0.03	2	0.02	1	1.5	30	40
		3	0.02				
		4	0.03				
		5	0.9				
		0	0.01				
		1	0.02				
a_3	0.06	2	0.02	1	1.5	30	40
		3	0.02				
		4	0.03				
		5	0.9				
		0	0.01				
		1	0.02				
a_4	0.02	2	0.02	1	1.5	30	40
		3	0.02				
		4	0.03				
		5	0.9				

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		0	0.01				
		1	0.02				
a_5	0.08	2	0.02	1	1.5	30	40
		3	0.02				
		4	0.03				
		5	0.9				
		0	0.01				
		1	0.02				
a_6	0.04	2	0.02	1	1.5	30	40
		3	0.02				
		4	0.03				
		5	0.9				
		0	0.01				
		1	0.02				
a_7	0.04	2	0.02	1	1.5	30	40
		3	0.02				
		4	0.03				
		5	0.9				
		0	0.01				
		1	0.02				
a_8	0.05	2	0.02	1	1.5	30	40
		3	0.02				
		4	0.03				
		5	0.9				

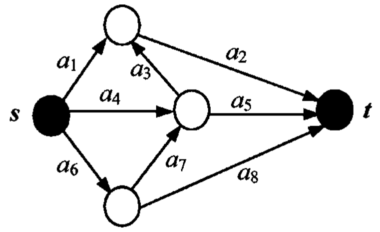

Fig. 3 A bridge network

Table 6. The results of Fig. 3 network

E	$R_{(d^1,d^2,C,E)}$	
	(d^1, d^2)=(5,6)	(d^1, d^2)=(4, 5)
1*	0.6395	0.8125
0.1	0	0
0.2	0.6395	0.8125
0.3	0.6395	0.8125
0.4	0.6395	0.8125
0.5	0.6395	0.8125
0.6	0.6395	0.8125
0.7	0.6395	0.8125
0.8	0.6395	0.8125
0.9	0.6395	0.8125

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6.2.3. Six-node example

The results of six-node network. The network and arc data are listed in Fig. 4 and Table 7, respectively. with transmission budget, K=600. There are five minimal paths: $mp_1=\{a_1, a_2, a_3\}$, $mp_2=\{a_1, a_4, a_8\}$, $mp_3=\{a_1, a_4, a_5, a_3\}$, $mp4=\{a_6, a_7, a_8\}$ and $mp_5=\{a_6, a_7, a_5, a_3\}$. If the demand is set to be (d^1, d^2), the results are listed in Table 8.

Table 7. The Arc Data six-node network

a_i	e_i	Capacity	Probability	w_i^1	w_i^2	c_i^1	c_i^2
		0	0.01				
		1	0.01				
a_1	0.05	2	0.02	1	1.5	20	30
		3	0.02				
		4	0.94				
		0	0.01				
		1	0.01				
a_2	0.03	2	0.02	1	1.5	20	25
		3	0.02				
		4	0.94				
		0	0.01				
		1	0.01				
a_3	0.06	2	0.02	1	2	30	40
		3	0.02				
		4	0.94				
		0	0.01				
		1	0.01				
a_4	0.02	2	0.02	1	2	40	60
		3	0.02				
		4	0.94				
		0	0.01				
		1	0.01				
a_5	0.08	2	0.02	1	1.5	20	30
		3	0.02				
		4	0.94				
		0	0.01				
		1	0.01				
a_6	0.04	2	0.02	1	1.5	20	25
		3	0.02				
		4	0.94				
		0	0.01				
		1	0.01				
a_7	0.04	2	0.02	1	2	30	50
		3	0.02				
		4	0.94				
		0	0.01				
		1	0.01				
a_8	0.05	2	0.02	1	2	40	60
		3	0.02				
		4	0.94				

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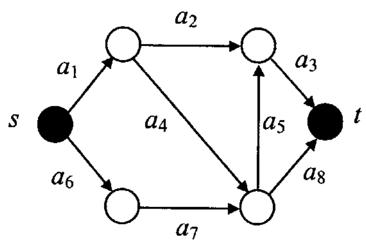

Fig. 4 A bridge network

Table 8. The results of Fig. 4 network

E	$R_{(d^1,d^2,C,E)}$	
	(d^1,d^2)=(4,2)	(d^1,d^2)=(3,1)
1*	0.7495	0.9469
0.1	0	0
0.2	0.7495	0.9469
0.3	0.7495	0.9469
0.4	0.7495	0.9469
0.5	0.7495	0.9469
0.6	0.7495	0.9469
0.7	0.7495	0.9469
0.8	0.7495	0.9469
0.9	0.7495	0.9469

7. Conclusions

This paper extends a multi-commodity reliability model with budget constraint and a transmission error rate. We evaluated the system reliability of a stochastic flow network to given demands d^1 and d^2 under the budget C and the tolerable error rate E constraints. We use an algorithm that is based on determining the set of all feasible solutions of the flow vector and generate the set of all lower boundary points for the given demands d^1 and d^2 under the budget C and the tolerable error rate E constraints, then calculate the reliability. We illustrate the use of the proposed formulation by calculating the reliability of a flow network to given sample network taken from literature. At the first example, we evaluated the Reliability evaluation of multi-commodity with Error constraint $R_{(d^1,d^2,E)}$. Then we evaluated Reliability evaluation of multi-commodity with Budget and Error constraints $R_{(d^1,d^2,C,E)}$.

Conflict of Interests: *The authors declare that they have no conflict of interests.*

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