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Original Article

Measuring and Improving the Reliability of Radar Design with Mixture Lifetime Distribution and Time Delay

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Abstract:

In this paper, a radar system consists of 3 non-identical and independent components are considered. Two components are linked together in parallel, and this subsystem is linked to a different component in series. The lifetimes of the system components are the mixture distribution with delayed time. Redundant and reduction techniques are provided to upgrade the system reliability. These techniques are hot, cold, and imperfect switch techniques. We calculate the mean time to failure, γ-fractiles and reliability equivalence factors to differentiate between various techniques. Simulation examples were used to illustrate the differences between different improving techniques. The simulation results indicated that the improved system performs better than the original one. Moreover, it showed that the best method to improve is by cold duplication.

Keywords: Reliability Improving Methods, Mixture Distributions, Exponential Distribution, Systems Performances, Equivalence Factors, Reduction Method, Duplication Methods.

1. Introduction

There are many techniques that can be used to upgrade system of the reliability. Reduction technique can be used to enhance some components quality by lowering their failure rates by a certain factor ρ, $0 < \rho < 1$. There are other techniques which can be used to upgrade systems reliability depending on increasing the redundancy of the components in the system such as hot, cold, and imperfect switch redundancy. These techniques are used to upgrade the efficiency of the system and increase the systems reliability. Råde (1993) has proposed four techniques to improve the reliability of systems and applied such techniques to various reliability systems.

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Thus, we shall make equivalence between the improved systems by equating the survival functions of the obtained systems at the level, say γ . Based on such process we could obtain the reliability equivalence factor at the level γ. Sarhan (2002) improved the performance of a series-parallel system using reliability equivalence factors. Sarhan (2004) introduced the reliability equivalence factors of a bridge network system. Sarhan and Mustafa (2006) proposed the reliability equivalence factors of a series system which consists of n independent and non-identical items. Mustafa (2007) obtained the reliability equivalence of n-component series systems with mixture failure rates. Mustafa, et al. (2007) derived three distinct forms of the reliability equivalence factors of a parallel-series system made up of four identical and independent components. Xia and Zhang (2007) examined the reliability equivalence factors of a parallel system with non-constant failure rates and gamma distribution. Sarhan et al. (2008) calculated equivalence factors of a parallel series system and presupposed that every component has independent exponentially distributed. Mustafa and El-Bassoiuny (2009) derived the reliability equivalence of some systems with mixture linear increasing failure rates. Mustafa et al. (2009) improved the performance of a series system with mixture failure rates using reliability equivalence factors. Mustafa (2009) upgraded the performance of some systems with mixture Weibull failure rates using reliability equivalence. Mustafa and El-Faheem (2011) obtained the reliability equivalence factors of a system with mixture distribution. Mustafa and El-Faheem (2012) improved the performance of a general parallel system with a mixture distribution and delayed time using reliability equivalence factors. Yousry and Al-Ohally (2013) improved the system reliability using exponentiated lifetime distribution. Mustafa and El-Faheem (2014) presented the reliability equivalence factors of a system with mixture distribution and delay time. Alghamdi and Percy (2017) upgraded the performance of a series–parallel system of components with exponentiated Weibull lifetimes using reliability equivalence factors. Ariffin and Shafie (2018) studied the failure time's parameter by mixture and standard Weibully distributed. Migdadi et al. (2019) improved the reliability performance of a general series-parallel systems using generalized exponential lifetime model. Alghazo et al. (2020) calculated the availability equivalence analysis for repairable bridge network system. Temraz (2020) calculated the reliability performance for series system using fuzzy equivalence factors. Yusuf et al. (2020) discussed the performance of multi computer systems consisting of active parallel homogeneous clients. Mohiuddin et al. (2021) studied the reliability of system using standby redundancy technique.

Maryam et al. (2021) derived the reliability of a system by Standby redundancy method. El-Faheem et al. (2022) improved the reliability performance for radar system using Rayleigh distribution. Zhang et al. (2022) improved the reliability optimization of parallel-series and series-parallel systems with statistically dependent components.

In this paper, analysis of the reliability equivalence factors of a complex system consisting of 3 independent and non-identical components is introduced. The reliability function of the original system is derived with a mixture distribution and delayed time. The reliability of the original system is improved according to reduction, hot, cold and imperfect techniques. The reliability equivalence factors are introduced to compare different system designs. Figure 1 illustrates the basic configuration of the radar system, see Roy Billinton and Ronald N. Allan 1992.

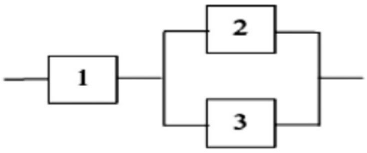

Figure 1: Radar system.

Let $f_1(t)$ be the probability density function (pdf) for the first stage with probability p and $f_2(t)$ be the probability density function (pdf) for the second stage with probability q. Therefore, a component will fail at the conclusion of both the first or second stage. Consequently, the failure time (pdf) for whole system:

$$f(t) = p\, f_1(t) + q\, f_2(t) \tag{1}$$

where $0 \le p, q \le 1, p + q = 1$. If the second failure mode happens after a delay time δ from the first failure mode, then:

$$f(t) = p\, f_1(t) + q\, f_2(t-\delta), \quad 0 \le \delta \le t \tag{2}$$

Assuming that the two kinds of mixture lifetime failure rates are $\lambda_i, i = 1, 2$, The time of system failure is indicated as

$$f(t) = \lambda_1 p\, exp\{-\lambda_1 t\} + \lambda_2 q\, exp\{-\lambda_2 (t-\delta)\} \tag{3}$$

The reliability function and hazard rate function may be calculated as follows:

$$\Re(t) = p\, exp\{-\lambda_1 t\} + q\, exp\{-\lambda_2 (t-\delta)\} \tag{4}$$

$$h(t) = \frac{\lambda_1 p\, exp\{-\lambda_1 t\} + \lambda_2 q\, exp\{-\lambda_2 (t-\delta)\}}{p\, exp\{-\lambda_1 t\} + q\, exp\{-\lambda_2 (t-\delta)\}} \tag{5}$$

From Equation (5) the hazard rate varies with time t.

2. Radar System

The radar system is said to be operating if one set from these system components sets {1, 2}, {1, 3} and {1, 2, 3} are in operating state, Barlow and Proschan (1981). The lifetime of every item is a mixture distribution exponential lifetime with delayed times. Let $\Re(t)$ represent the radar system of the reliability function (RF). $\Re(t)$ may be described following:

$$\Re(t) = \Re_1(t)[\Re_2(t) + \Re_3(t) - \Re_2(t)\,\Re_3(t)] \tag{6}$$

Mean time to failure (MTTF) is given by:

$$MTTF = \int_0^\infty \Re(t)\, dt \tag{7}$$

MTTF may be calculated numerically with program such as Mathematica.

3. Methods of Improved system

Four different improvement techniques may be used to upgrade the reliability of the system.

3.1. Reduction method:

Let $\Re_{\Lambda,\rho}(t)$ be the reliability function of the upgraded system when the failure rate of two type of mixture lifetimes of a set Λ of the system components when Λ= {1}, {2}, {1, 2}, {2, 3} and {1, 2, 3} are decreased by the factor ρ_i , $0 < \rho_i < 1, i = 1, 2$. The function $\Re_{\Lambda,\rho}(t)$ is given by:

$$\Re_\rho(t) = p\, exp\{-\rho_1 \lambda_1 t\} + q\, exp\{-\rho_2 \lambda_2 (t-\delta)\} \tag{8}$$

(i) At $\Lambda = \{1\}$,

$$\Re_{\{1\},\rho}(t) = \Re_{1,\rho}(t)\big[\Re_3(t) - \Re_2(t)\big(1 - \Re_3(t)\big)\big] \tag{9}$$

(ii) At $\Lambda = \{2\}$,

$$\mathfrak{R}_{\{2\},\rho}(t) = \mathfrak{R}_1(t)\big[\mathfrak{R}_3(t) + \mathfrak{R}_{2,\rho}(t)\big(1 - \mathfrak{R}_3(t)\big)\big] \tag{10}$$

(iii) At $\Lambda = \{1, 2\}$,

$$\mathfrak{R}_{\{1,2\},\rho}(t) = \mathfrak{R}_{1,\rho}(t)\big[\mathfrak{R}_3(t) + \mathfrak{R}_{2,\rho}(t)\big(1 - \mathfrak{R}_3(t)\big)\big] \tag{11}$$

(iv) At $\Lambda = \{2, 3\}$,

$$\mathfrak{R}_{\{2,3\},\rho}(t) = \mathfrak{R}_1(t)\left[\mathfrak{R}_{3,\rho}(t) + \mathfrak{R}_{2,\rho}(t)\left(1 - \mathfrak{R}_{3,\rho}(t)\right)\right] \tag{12}$$

(v) At $\Lambda = \{1, 2, 3\}$,

$$\mathfrak{R}_{\{1,2,3\},\rho}(t) = \mathfrak{R}_{1,\rho}(t)\left[\mathfrak{R}_{3,\rho}(t) + \mathfrak{R}_{2,\rho}(t)\left(1 - \mathfrak{R}_{3,\rho}(t)\right)\right] \tag{13}$$

MTTF for the upgraded system by using reduction technique, $MTTF_{\Lambda,\rho}$, may be calculated by

$$MTTF_{\Lambda,\rho} = \int_0^\infty \mathfrak{R}_{\Lambda,\rho}(t)\, dt. \tag{14}$$

The integration in Equation (14) may be computed numerically using programs for computing numbers, such as Mathematica.

3.2. Hot Duplication Method

Let $\mathfrak{R}_\Lambda^{\mathcal{H}}(t)$ be the reliability function of the upgraded system obtained by assuming hot duplications of a set Λ of system components. Obtaining the function $\mathfrak{R}^{\mathcal{H}}(t)$ by:

$$\mathfrak{R}^{\mathcal{H}}(t) = \big(2 - \mathfrak{R}(t)\big)\, \mathfrak{R}(t) \tag{15}$$

(i) At $\Lambda = \{1\}$,

$$\mathfrak{R}_{\{1\}}^{\mathcal{H}}(t) = \mathfrak{R}_1^{\mathcal{H}}(t)\big[\mathfrak{R}_3(t) - \mathfrak{R}_2(t)\big(1 - \mathfrak{R}_3(t)\big)\big] \tag{16}$$

(ii) At $\Lambda = \{2\}$,

$$\mathfrak{R}_{\{2\}}^{\mathcal{H}}(t) = \mathfrak{R}_1(t)\big[\mathfrak{R}_3(t) - \mathfrak{R}_2^{\mathcal{H}}(t)\big(1 - \mathfrak{R}_3(t)\big)\big] \tag{17}$$

(iii) At $\Lambda = \{1, 2\}$,

$$\mathfrak{R}_{\{1,2\}}^{\mathcal{H}}(t) = \mathfrak{R}_1^{\mathcal{H}}(t)\big[\mathfrak{R}_3(t) - \mathfrak{R}_2^{\mathcal{H}}(t)\big(1 - \mathfrak{R}_3(t)\big)\big] \tag{18}$$

(iv) At $\Lambda = \{2, 3\}$,

$$\mathfrak{R}_{\{2,3\}}^{\mathcal{H}}(t) = \mathfrak{R}_1(t)\left[\mathfrak{R}_3^{\mathcal{H}}(t) + \mathfrak{R}_2^{\mathcal{H}}(t)\left(1 - \mathfrak{R}_3^{\mathcal{H}}(t)\right)\right] \tag{19}$$

(v) At $\Lambda = \{1, 2, 3\}$,

$$\mathfrak{R}_{\{1,2,3\}}^{\mathcal{H}}(t) = \mathfrak{R}_1^{\mathcal{H}}(t)\left[\mathfrak{R}_3^{\mathcal{H}}(t) + \mathfrak{R}_2^{\mathcal{H}}(t)\left(1 - \mathfrak{R}_3^{\mathcal{H}}(t)\right)\right] \tag{20}$$

MTTF for the upgraded system by using the hot technique, $MTTF_\Lambda^{\mathcal{H}}$, may be calculated by

$$\mathrm{MTTF}_\Lambda^{\mathcal{H}} = \int_0^\infty \mathfrak{R}_\Lambda^{\mathcal{H}}(t)\, dt. \tag{21}$$

The integration in Equation (21) may be calculated numerically with program such as Mathematica.

4.3. Cold duplication method

Let $\mathfrak{R}_\Lambda^{\mathcal{C}}(t)$ be the reliability function of the upgraded system by assuming cold duplications of the components belong to the set Λ of the system components. When the CDM improves the component *i*, *i* = 1, 2, 3, $\mathfrak{R}^{\mathcal{C}}(t)$ may be provided as follows:

$$\mathfrak{R}^{\mathcal{C}}(t) = \mathfrak{R}(t) + \int_0^t f(x)\, \mathfrak{R}(t - x) dx \tag{22}$$

Equation (22) can be assigned to the values of λ_i and δ, but it cannot be obtained in general.

(i) At $\Lambda = \{1\}$,

$$\mathfrak{R}^{\mathcal{C}}_{\{1\}}(t) = \mathfrak{R}^{\mathcal{C}}_1(t)\left[\mathfrak{R}_3(t) - \mathfrak{R}_2(t)\left(1 - \mathfrak{R}_3(t)\right)\right] \tag{23}$$

(ii) At $\Lambda = \{2\}$,

$$\mathfrak{R}^{\mathcal{C}}_{\{2\}}(t) = \mathfrak{R}_1(t)\left[\mathfrak{R}_3(t) - \mathfrak{R}^{\mathcal{C}}_2(t)\left(1 - \mathfrak{R}_3(t)\right)\right] \tag{24}$$

(iii) At $\Lambda = \{1, 2\}$,

$$\mathfrak{R}^{\mathcal{C}}_{\{1,2\}}(t) = \mathfrak{R}^{\mathcal{C}}_1(t)\left[\mathfrak{R}_3(t) - \mathfrak{R}^{\mathcal{C}}_2(t)\left(1 - \mathfrak{R}_3(t)\right)\right] \tag{25}$$

(iv) At $\Lambda = \{2, 3\}$,

$$\mathfrak{R}^{\mathcal{C}}_{\{2,3\}}(t) = \mathfrak{R}_1(t)\left[\mathfrak{R}^{\mathcal{C}}_3(t) + \mathfrak{R}^{\mathcal{C}}_2(t)\left(1 - \mathfrak{R}^{\mathcal{C}}_3(t)\right)\right] \tag{26}$$

(v) At $\Lambda = \{1, 2, 3\}$,

$$\mathfrak{R}^{\mathcal{C}}_{\{1,2,3\}}(t) = \mathfrak{R}^{\mathcal{C}}_1(t)\left[\mathfrak{R}^{\mathcal{C}}_3(t) + \mathfrak{R}^{\mathcal{C}}_2(t)\left(1 - \mathfrak{R}^{\mathcal{C}}_3(t)\right)\right] \tag{27}$$

MTTF for the upgraded system by using cold technique, $MTTF^{\mathcal{C}}_{\Lambda}$, can be calculated by

$$MTTF^{\mathcal{C}}_{\Lambda} = \int_0^{\infty} \mathfrak{R}^{\mathcal{C}}_{\Lambda}(t)\, dt. \tag{28}$$

The integration in Equation (28) may be calculated numerically with program such as Mathematica.

4.4. Imperfect switch technique

In such technique, it is assumed that the component is linked by a cold redundant standby component through a random switch having a fixed failure rate $\beta_i, i = 1, 2$. So, the RF for the switch is

$$\mathfrak{R}_{sw}(t) = p\, exp\{-\beta_1 t\} + q\, exp\{-\beta_2 (t - \delta)\}. \tag{29}$$

Let $\mathfrak{R}^{\mathcal{I}}_{\Lambda}(t)$ denote the reliability function system of the component, that be upgraded by using imperfect switch duplication technique. The following is the provided for the function $\mathfrak{R}^{\mathcal{I}}(t)$

$$\mathfrak{R}^{\mathcal{I}}(t) = \mathfrak{R}(t) + \int_0^t f(x)\, \mathfrak{R}_{sw}(t - x)\, \mathfrak{R}(t - x) dx \tag{30}$$

Equation (30) can be assigned to the values of λ_i, β_i and δ, but it cannot be obtained in general.

(i) At $\Lambda = \{1\}$,

$$\mathfrak{R}^{\mathcal{I}}_{\{1\}}(t) = \mathfrak{R}^{\mathcal{I}}_1(t)\left[\mathfrak{R}_3(t) - \mathfrak{R}_2(t)\left(1 - \mathfrak{R}_3(t)\right)\right] \tag{31}$$

(ii) At $\Lambda = \{2\}$,

$$\mathfrak{R}^{\mathcal{I}}_{\{2\}}(t) = \mathfrak{R}_1(t)\left[\mathfrak{R}_3(t) - \mathfrak{R}^{\mathcal{I}}_2(t)\left(1 - \mathfrak{R}_3(t)\right)\right] \tag{32}$$

(iii) At $\Lambda = \{1, 2\}$,

$$\mathfrak{R}^{\mathcal{I}}_{\{1,2\}}(t) = \mathfrak{R}^{\mathcal{I}}_1(t)\left[\mathfrak{R}_3(t) - \mathfrak{R}^{\mathcal{I}}_2(t)\left(1 - \mathfrak{R}_3(t)\right)\right] \tag{33}$$

(iv) At $\Lambda = \{2, 3\}$,

$$\mathfrak{R}^{\mathcal{I}}_{\{2,3\}}(t) = \mathfrak{R}_1(t)\left[\mathfrak{R}^{\mathcal{I}}_3(t) + \mathfrak{R}^{\mathcal{I}}_2(t)\left(1 - \mathfrak{R}^{\mathcal{I}}_3(t)\right)\right] \tag{34}$$

(v) At $\Lambda = \{1, 2, 3\}$,

$$\mathfrak{R}^{\mathcal{I}}_{\{1,2,3\}}(t) = \mathfrak{R}^{\mathcal{I}}_1(t)\left[\mathfrak{R}^{\mathcal{I}}_3(t) + \mathfrak{R}^{\mathcal{I}}_2(t)\left(1 - \mathfrak{R}^{\mathcal{I}}_3(t)\right)\right] \tag{35}$$

MTTF for the upgraded system by using the imperfect technique, $MTTF^{\mathcal{I}}_{\Lambda}$, can be calculated by

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$$MTTF_{\Lambda}^{\mathcal{I}} = \int_0^{\infty} \Re_{\Lambda}^{\mathcal{I}}(t)\, dt. \qquad (36)$$

The integration in Equation (36) may be calculated numerically with program such as Mathematica.

5. γ- Fractiles

Let $L(\gamma)$ and $L_{\Lambda}^{\mathcal{D}}(\gamma)$ be the γ-fractile of the original and upgraded systems. γ-fractiles is given by:

$$\Re\big(L(\gamma)\big) = \gamma, \qquad \Re_{\Lambda}^{\mathcal{D}}\left(L_{B}^{\mathcal{D}}(\gamma)\right) = \gamma, \quad \mathcal{D} = \mathcal{H}, \mathcal{C} \text{ and } \mathcal{I}. \qquad (37)$$

First part of Equation (37) with Equation (6), give γ-fractiles for the basic system as a solution for the next equation w. r. t. L.

$$\Re_1\big(L(\gamma)\big)\left[\Re_2\big(L(\gamma)\big) + \Re_3\big(L(\gamma)\big) - \Re_2\big(L(\gamma)\big)\,\Re_3\big(L(\gamma)\big)\right] = \gamma \qquad (38)$$

By the same way, we can obtained the γ-fractile for the upgraded system by using the second equation of (37) and Equations (16-20), (23-27) and (31-35), for $\mathcal{D} = \mathcal{H}, \mathcal{C}$ and $\mathcal{I}$, respectively. The equations for γ-fractiles may be calculated numerically with program such as Mathematica.

6. Reliability Equivalence Factors (REF)

We will calculate (REF), when the mixture failure lifetime of the system component is reduced by the factor ρ_i , $0 < \rho_i < 1, i = 1, 2$. The factor $\rho_{\Lambda}^{\mathcal{D}}(\gamma)$ is given by:

$$\Re_{\Lambda}^{\mathcal{D}}(t) = \Re_{\Lambda,\rho}(t) = \gamma\ , \ \ \mathcal{D} = \mathcal{H}, \mathcal{C} \text{ and } \mathcal{I}. \qquad (39)$$

- Substituting from (9)-(13) and (16)-(20) in (39), at $\mathcal{D} = \mathcal{H}$, the hot REFs, $\rho_{\Lambda}^{\mathcal{H}}(\gamma)$, can be obtained.
- Also, substituting from (9)-(13) and (31)-(35) in (39), at $\mathcal{D} = \mathcal{I}$, the imperfect REFs, $\rho_{\Lambda}^{\mathcal{I}}(\gamma)$ can be calculated.
- Substituting from (9)-(13) and (23)-(27) into (39), the cold REFs, $\rho_{\Lambda}^{\mathcal{C}}(\gamma)$, can be calculated at $\mathcal{D} = \mathcal{C}$.

The $\rho_{\Lambda}^{\mathcal{D}}(\gamma)$ may be calculated numerically with program such as Mathematica.

7. Numerical simulations and discussions

Under the following presumptions, the REFs of a radar system with a single component and two non-identical mixing lifetimes can be calculated:

1) The lifetime mixture failure rates are $\lambda_1 = 0.07$ and $\lambda_2 = 0.09$.
2) The probability $p = 0.40$, and $q = 0.60$.
3) In the ISDM are $\beta_1 = 0.01$ and, $\beta_2 = 0.01$.
4) The delayed time $\delta = 0.03$.

We get $MTTF = 8.21941$ and Table 1 contains the $MTTF_{\Lambda}^{\mathcal{D}}$, Λ of the upgraded systems assuming HDM, CDM, IDM.

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Table 1: The $MTTF_{\Lambda}^{\mathcal{D}}$, $\mathcal{D} = \mathcal{H}, \mathcal{C}, \mathcal{J}$ and Λ.

Λ	$MTTF_{\Lambda}^{\mathcal{H}}$	$MTTF_{\Lambda}^{\mathcal{J}}$	$MTTF_{\Lambda}^{\mathcal{C}}$
{1}	11.3098	12.7327	13.0397
{2}	9.25215	9.81569	9.94214
{1,2}	12.9640	16.1045	16.9354
{2,3}	9.87369	10.5970	10.7468
{1,2,3}	14.0011	17.9382	18.9982

From Table 1, therefore, we can say:

$MTTF < MTTF_{\Lambda}^{\mathcal{H}} < MTTF_{\Lambda}^{\mathcal{J}} < MTTF_{\Lambda}^{\mathcal{C}}$, for all Λ.

The $L(\gamma)$, $L_{\Lambda}^{\mathcal{D}}(\gamma)$ and $\rho_{\Lambda}^{\mathcal{D}}(\gamma)$, for $\mathcal{D} = \mathcal{H}, \mathcal{C}, \mathcal{J}$, with $\gamma = 0.1, 0.2, \cdots, 0.9$ are contained in tables 2 and 3.

Table 2: The $L(\gamma)$, $L_{\Lambda}^{\mathcal{D}}(\gamma)$ for $\mathcal{D} = \mathcal{H}, \mathcal{C}, \mathcal{J}$ and $\Lambda = \{1\}, \{2\}$ and $\{1,2\}$.

γ	$L(\gamma)$	$\Lambda = \{1\}$			$\Lambda = \{2\}$			$\Lambda = \{1,2\}$		
		$L^{\mathcal{H}}$	$L^{\mathcal{J}}$	$L^{\mathcal{C}}$	$L^{\mathcal{H}}$	$L^{\mathcal{J}}$	$L^{\mathcal{C}}$	$L^{\mathcal{H}}$	$L^{\mathcal{J}}$	$L^{\mathcal{C}}$
0.1	17.696	21.686	24.316	24.923	19.709	21.182	21.526	23.968	29.357	30.875
0.2	12.992	16.777	18.881	19.341	14.722	15.739	15.959	18.906	23.370	24.569
0.3	10.138	13.743	15.500	15.872	11.619	12.344	12.492	15.737	19.577	20.578
0.4	8.0349	11.462	12.949	13.254	9.2773	9.7848	9.8829	13.325	16.664	17.516
0.5	6.3337	9.5741	10.830	11.082	7.3389	7.6759	7.7374	11.301	14.204	14.930
0.6	4.8764	7.9080	8.9570	9.1619	5.6416	5.8447	5.8795	9.4869	11.986	12.600
0.7	3.5737	6.3568	7.2098	7.3721	4.0971	4.2003	4.2167	7.7663	9.8704	10.378
0.8	2.3655	4.8244	5.4820	5.6029	2.6552	2.6932	2.6985	6.0254	7.7172	8.1174
0.9	1.2010	3.1577	3.6037	3.6811	1.2935	1.3004	1.3011	4.0653	5.2772	5.5556

Table 3: The $L(\gamma)$, $L^{\Lambda\mathcal{D}}(\gamma)$ for $\mathcal{D} = \mathcal{H}, \mathcal{C}, \mathcal{J}$ and $\Lambda = \{2,3\}$ and $\{1,2,3\}$

γ	$L(\gamma)$	$\Lambda = \{2,3\}$			$\Lambda = \{1,2,3\}$		
		$L^{\mathcal{H}}$	$L^{\mathcal{J}}$	$L^{\mathcal{C}}$	$L^{\mathcal{H}}$	$L^{\mathcal{J}}$	$L^{\mathcal{C}}$
0.1	17.696	21.096	23.175	23.626	25.552	32.243	34.147
0.2	12.992	15.873	17.245	17.513	20.358	25.974	27.509
0.3	10.138	12.559	13.469	13.631	17.071	21.941	23.237
0.4	8.0349	10.015	10.587	10.680	14.542	18.799	19.911
0.5	6.3337	7.8791	8.2051	8.2522	12.395	16.106	17.058
0.6	4.8764	5.9949	6.1520	6.1722	10.447	13.641	14.447
0.7	3.5737	4.2873	4.3449	4.3514	8.5729	11.251	11.915
0.8	2.3655	2.7259	2.7389	2.7402	6.6474	8.7753	9.2937
0.9	1.2010	1.3042	1.3052	1.3053	4.4461	5.9235	6.2732

Figures 2-6 show the reliability function for the original and upgrade system for various sets of Λ and techniques.

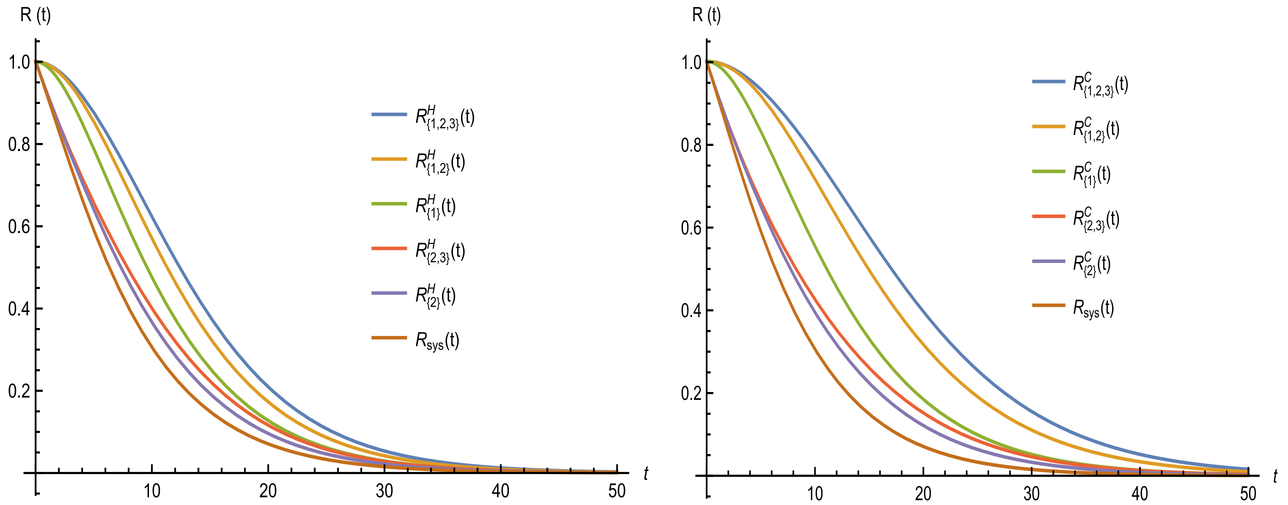

Figure 2: The $\Re(t), \Re_{\Lambda}^{\mathcal{D}}(t)$, for $\mathcal{D} = \mathcal{H}, \mathcal{C}$, and Λ .

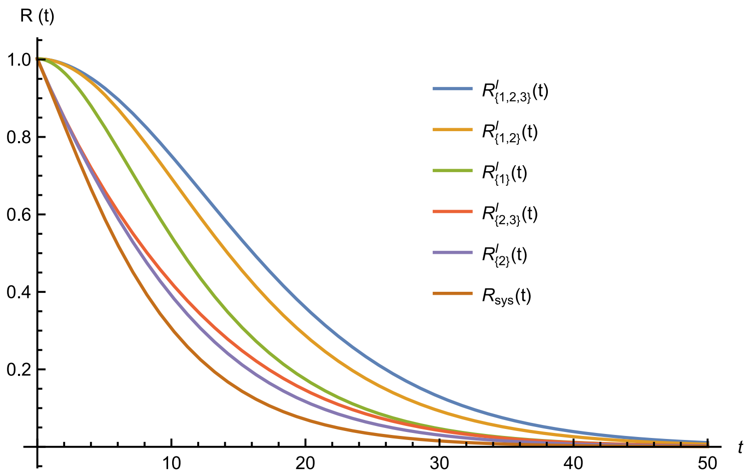

Figure 3: The $\Re(t), \Re_{\Lambda}^{\mathcal{D}}(t)$, when Λ.

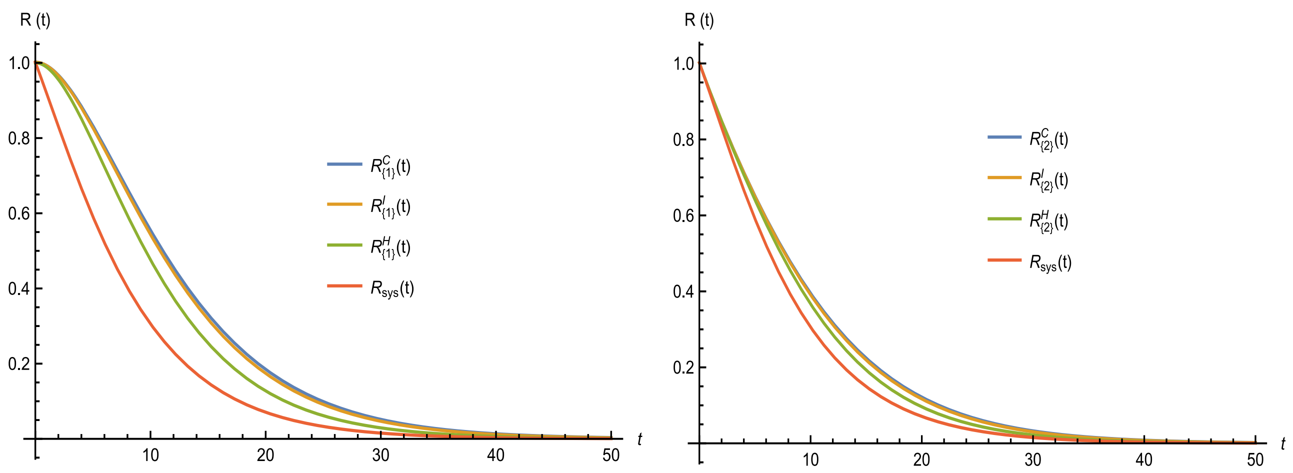

Figure 4: The $\Re(t), \Re_{\Lambda}^{\mathcal{D}}(t)$, for $\mathcal{D} = \mathcal{H}, \mathcal{C}, \mathcal{I}$ and $\Lambda = \{1\}$ and $\{2\}$.

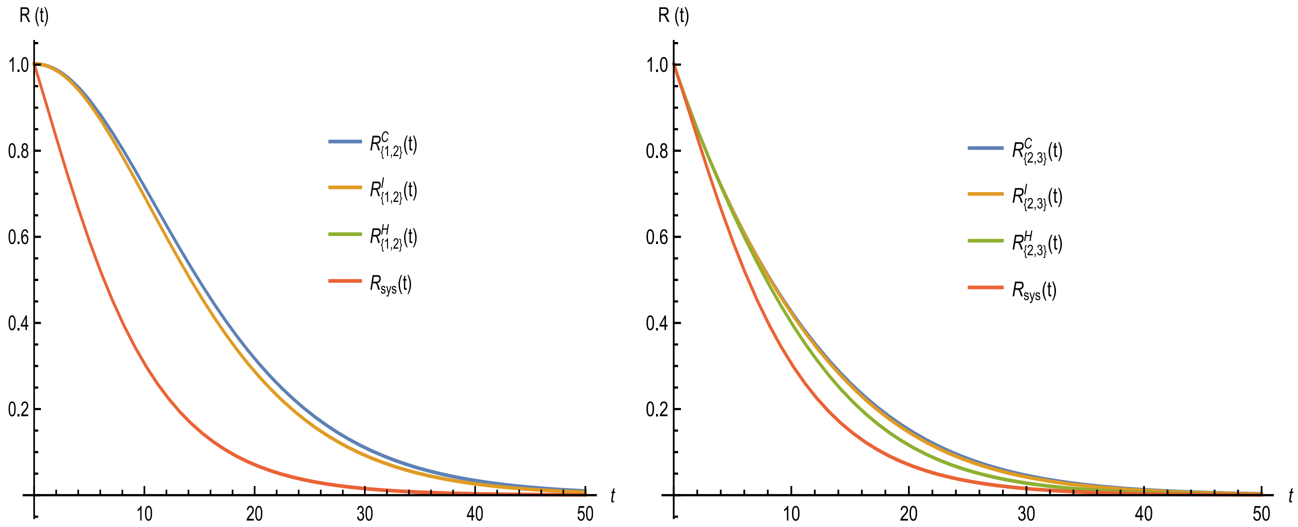

Figure 5: The $\mathfrak{R}(t), \mathfrak{R}_{\Lambda}^{\mathcal{D}}(t)$, for $\mathcal{D} = \mathcal{H}, \mathcal{C}, \mathcal{J}$ and $\Lambda = \{1,2\}$ and $\{2,3\}$.

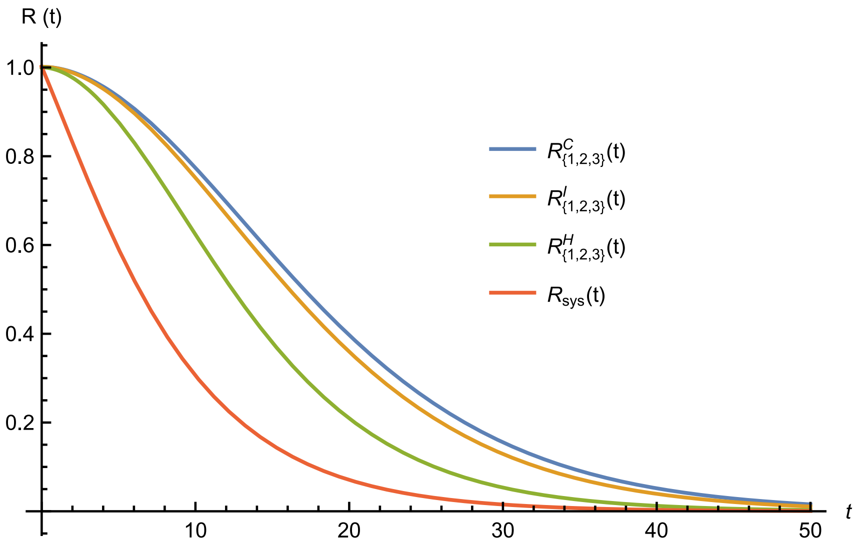

Figure 6: The $\mathfrak{R}(t), \mathfrak{R}_{\{1,2,3\}}^{\mathcal{D}}(t)$, for $\mathcal{D} = \mathcal{H}, \mathcal{C}$ *and* $\mathcal{J}$.

From Figures 2 – 6 and Tables 2, 3, we may conclude that:

1. $\mathfrak{R}(t) < \mathfrak{R}_{\Lambda}^{\mathcal{H}}(t) < \mathfrak{R}_{\Lambda}^{\mathcal{J}}(t) < \mathfrak{R}_{\Lambda}^{\mathcal{C}}(t)$ for all Λ.
2. $\mathfrak{R}(t) < \mathfrak{R}_{\{2\}}^{\mathcal{D}}(t) < \mathfrak{R}_{\{2,3\}}^{\mathcal{D}}(t) < \mathfrak{R}_{\{1\}}^{\mathcal{D}}(t) < \mathfrak{R}_{\{1,2\}}^{\mathcal{D}}(t) < \mathfrak{R}_{\{1,2,3\}}^{\mathcal{D}}(t)$ For all $\mathcal{D} = \mathcal{H}, \mathcal{J}$ and $\mathcal{C}$.
3. $L(\gamma) < L_{\Lambda}^{\mathcal{H}}(\gamma) < L_{\Lambda}^{\mathcal{J}}(\gamma) < L_{\Lambda}^{\mathcal{C}}(\gamma)$, for all Λ.
4. $L(\gamma) < L_{\{2\}}^{\mathcal{D}}(\gamma) < L_{\{2,3\}}^{\mathcal{D}}(\gamma) < L_{\{1\}}^{\mathcal{D}}(\gamma) < L_{\{1,2\}}^{\mathcal{D}}(\gamma) < L_{\{1,2,3\}}^{\mathcal{D}}(\gamma)$ For all $\mathcal{D} = \mathcal{H}, \mathcal{J}, \mathcal{C}$.
5. As γ increases, the MTTF, the γ-fractiles of the improved systems increase too.

REFs are presented in Tables 4 and 5, for some sets Λ.

Table 4: The $\rho_{\Lambda}^{\mathcal{D}}(\gamma)$ for $\mathcal{D} = \mathcal{H}, \mathcal{C}, \mathcal{J}$ and $\Lambda = \{1\}, \{2\}$ and $\{1,2\}$.

γ	Λ	{1}			{2}			{1,2}		
		$\rho^{\mathcal{H}}$	$\rho^{\mathcal{J}}$	$\rho^{\mathcal{C}}$	$\rho^{\mathcal{H}}$	$\rho^{\mathcal{J}}$	$\rho^{\mathcal{C}}$	$\rho^{\mathcal{H}}$	$\rho^{\mathcal{J}}$	$\rho^{\mathcal{C}}$
0.1	{1}	0.6534	0.4849	0.4508	0.8084	0.6903	0.6650	0.5052	0.2437	0.1862
	{2}	0.4043	0.2050	0.1689	0.6299	0.4537	0.4195	0.2270	–	–
	{1,2}	0.7690	0.6653	0.6450	0.8695	0.7925	0.7763	0.6774	0.5275	0.4966
	{2,3}	0.5951	0.3878	0.3433	0.7777	0.6389	0.6089	0.4137	0.0003	–
	{1,2,3}	0.8159	0.7276	0.7098	0.8978	0.8353	0.8219	0.7381	0.6025	0.5729
0.2	{1}	0.5890	0.4277	0.3968	0.7877	0.6833	0.6624	0.4261	0.1757	0.1232
	{2}	0.2563	0.0669	0.0346	0.5552	0.3875	0.3569	0.0650	–	–
	{1,2}	0.7206	0.6204	0.6019	0.8517	0.7819	0.7681	0.6194	0.4759	0.4478
	{2,3}	0.4632	0.2105	0.1177	0.7288	0.5918	0.5638	0.2072	0.0002	–
	{1,2,3}	0.7742	0.6878	0.6714	0.8824	0.8253	0.8139	0.6869	0.5556	0.5285
0.3	{1}	0.5372	0.3841	0.3557	0.7781	–	–	0.3659	0.1272	0.0786
	{2}	0.1185	0.0587	0.0298	0.4934	0.3355	0.3079	–	–	–
	{1,2}	0.6795	0.5835	0.5663	0.8411	0.7788	0.7671	0.5725	0.4359	0.4100
	{2,3}	0.3047	–	–	0.6863	0.5521	0.5256	0.1512	0.0001	–
	{1,2,3}	0.7373	0.6536	0.6383	0.8724	0.8210	0.8113	0.6438	0.5174	0.4922
0.4	{1}	0.4899	0.3455	0.3195	0.7758	–	–	0.3136	0.0874	0.0423
	{2}	0.0924	–	0.0154	0.4354	0.2887	0.2639	–	–	–
	{1,2}	0.6397	0.5485	0.5327	0.8352	0.7705	0.6528	0.5291	0.4000	0.3761
	{2,3}	0.0010	–	–	0.6863	0.5135	0.4885	0.0102	0.0000	–
	{1,2,3}	0.7005	0.6199	0.6056	0.8658	0.8208	0.7048	0.6024	0.4816	0.4581
0.5	{1}	0.4434	0.3088	0.2850	0.7751	–	–	0.2651	0.0525	0.01079
	{2}	–	–	–	0.3774	0.2435	0.2217	–	–	–
	{1,2}	0.5986	0.5128	0.4982	0.8339	0.6578	–	0.4862	0.3652	0.3433
	{2,3}	0.0001	–	–	0.6001	0.4732	0.3486	0.0023	–	–
	{1,2,3}	0.6609	0.5841	0.5708	0.8627	–	–	0.5597	0.4452	0.4235
0.6	{1}	0.3956	0.2718	0.2504	–	–	–	0.2182	0.0208	–
	{2}	–	–	–	0.3167	0.1980	0.1794	–	–	–
	{1,2}	0.5533	0.4739	0.4607	0.8320	–	–	0.4411	0.3294	0.3096
	{2,3}	–	–	–	0.5500	0.3546	0.3392	0.0010	–	–
	{1,2,3}	0.6157	0.5434	0.5312	0.8639	–	–	0.5130	0.4059	0.3861
0.7	{1}	0.3436	0.2326	0.2139	–	–	–	0.1712	0.0085	–

	{2}	–	–	–	0.2510	0.1505	0.1355	–	–	–
	{1,2}	0.5006	0.4288	0.4171	–	–	–	0.3911	0.2904	0.2728
	{2,3}	–	–	–	0.4904	0.3320	0.3162	0.0019	–	–
	{1,2,3}	0.5609	0.4943	0.4834	0.8549	–	–	0.4588	0.3608	0.3431
0.8	{1}	0.2834	0.1881	0.1725	–	–	–	0.1221	–	–
	{2}	–	–	–	0.1777	0.0992	0.0885	–	–	–
	{1,2}	0.4339	0.3718	0.3621	–	–	–	0.3314	0.2445	0.2297
	{2,3}	–	–	–	0.4139	0.3068	0.2718	0.0009	–	–
	{1,2,3}	0.4883	0.4295	0.4202	–	–	–	0.3906	0.3047	0.2897
0.9	{1}	0.2048	0.1312	0.1199	–	–	–	0.0682	–	–
	{2}	–	–	–	0.0945	0.0394	0.0336	–	–	–
	{1,2}	0.3360	0.2878	0.2807	–	–	–	0.2495	0.1824	0.1714
	{2,3}	–	–	–	0.3036	0.1952	0.1802	0.0000	–	–
	{1,2,3}	0.3765	0.3296	0.3227	–	–	–	0.2920	0.2247	0.2134

Table 5: The $\rho_\Lambda^{\mathcal{D}}(\gamma)$ for $\mathcal{D} = \mathcal{H}, \mathcal{C}, \mathcal{I}$ and $\Lambda = \{2,3\}$ and $\{1,2,3\}$.

γ	Λ	{2,3}			{1,2,3}		
		$\rho^{\mathcal{H}}$	$\rho^{\mathcal{I}}$	$\rho^{\mathcal{C}}$	$\rho^{\mathcal{H}}$	$\rho^{\mathcal{I}}$	$\rho^{\mathcal{C}}$
0.1	{1}	0.6968	0.5534	0.5256	0.4173	0.1389	0.0792
	{2}	0.4626	0.2815	0.2497	0.1345	–	–
	{1,2}	0.7967	0.7068	0.6897	0.6254	0.4717	0.4410
	{2,3}	0.6466	0.4740	0.4394	0.1811	–	–
	{1,2,3}	0.8387	0.7634	0.7488	0.6924	0.5486	0.5180
0.2	{1}	0.6705	0.5500	0.5285	0.3333	0.0675	0.0130
	{2}	0.3686	0.2068	0.1807	–	–	–
	{1,2}	0.7735	0.6958	0.6824	0.5645	0.4187	0.3909
	{2,3}	0.5747	0.4076	0.3761	–	–	–
	{1,2,3}	0.8183	0.7531	0.7416	0.6379	0.4999	0.4719
0.3	{1}	0.6623	0.5443	–	0.2718	0.0191	NA
	{2}	0.2957	0.1543	0.1328	–	–	–
	{1,2}	0.7619	–	–	0.5169	0.3791	0.3537
	{2,3}	0.5137	0.3894	0.2371	–	–	–
	{1,2,3}	0.8069	0.7524	0.7301	0.5934	0.4616	0.4358
0.4	{1}	–	–	–	0.2205	0.0187	–
	{2}	0.2322	0.1123	0.0954	–	–	–

	{1,2}	0.7585	–	–	0.4741	0.3448	0.3214
	{2,3}	0.3294	0.3058	0.2229	–	–	–
	{1,2,3}	0.8019	–	–	0.5519	0.4268	0.4030
0.5	{1}	–	–	–	0.1749	–	–
	{2}	0.1743	0.0778	0.0652	–	–	–
	{1,2}	–	–	–	0.433	0.3125	0.2912
	{2,3}	0.3135	0.2555	0.2005	–	–	–
	{1,2,3}	–	–	–	0.5102	0.3925	0.3706
0.6	{1}	–	–	–	0.1330	–	–
	{2}	0.1213	0.0495	0.0408	–	–	–
	{1,2}	–	–	–	0.3910	0.2805	0.2612
	{2,3}	0.3302	0.2061	0.1703	–	–	–
	{1,2,3}	–	–	–	0.4658	0.3566	0.3366
0.7	{1}	–	–	–	0.0939	–	–
	{2}	0.0741	0.0272	0.0221	–	–	–
	{1,2}	–	–	–	0.3460	0.2468	0.2297
	{2,3}	0.2378	0.1549	0.1327	–	–	–
	{1,2,3}	–	–	–	0.4156	0.3164	0.2988
0.8	{1}	–	–	–	0.0571	–	–
	{2}	0.0355	0.0111	0.0089	–	–	–
	{1,2}	–	–	–	0.2938	0.2086	0.1942
	{2,3}	0.1816	0.1009	0.0881	–	–	–
	{1,2,3}	–	–	–	0.3539	0.2679	0.2529
0.9	{1}	–	–	–	0.0236	–	–
	{2}	0.0095	0.0016	0.0012	–	–	–
	{1,2}	–	–	–	0.2242	0.1585	0.1478
	{2,3}	0.0952	0.0396	0.0338	–	–	–
	{1,2,3}	–	–	–	0.2669	0.2001	0.1889

Tables 4 – 5 allow for the conclusion that:

(1) Upgrading the set $\Lambda = \{1\}$ by HDM, L (0.1) will increase from 17.696 to 21.686, see Table 2. Similar impact on L (0.1) can be achieved by lowering the set's failure rate: (i) $\Lambda = \{1\}$, by the same factor $\rho = 0.6534$, (ii) $\Lambda = \{2\}$ by $\rho = 0.4075$, (iii) $\Lambda = \{1, 2\}$, by $\rho = 0.7690$, (iv) $\Lambda = \{2, 3\}$ by $\rho = 0.5951$, (v) $\Lambda = \{1, 2, 3\}$, by $\rho = 0.8159$, see Table 4.

(2) Upgrading the set $\Lambda = \{1\}$, by ISDM, L (0.1) will increase from 17.696 to 24.316, see Table 2. Similar impact on L (0.1) can be achieved by lowering the set's failure rate: (i) $\Lambda = \{1\}$

by ρ =0.4849, (ii) Λ = {2}, by ρ = 0.2050, (iii) Λ ={1,2} by ρ = 0.6653 (iv) Λ = {2, 3}by ρ = 0.3878 , (v) Λ ={1,2,3} by the same factor ρ = 0.7276, see Table 4.

(3) Upgrading the set Λ = {1}, by CDM, L (0.1) will increase from 17.696 to 24.923, see Table 2. Similar impact on L (0.1) can be achieved by lowering the set's failure rate: (i) Λ = {1} by ρ =0.4508, (ii) Λ = {2} by ρ = 0.1689, (iii) Λ = {1, 2} by ρ = 0.6450 (iv) Λ = {2,3} by ρ = 0.3433, (v) Λ = {1,2,3} by ρ = 0.7098, see Table 4.

(4) The remaining results shown in Tables 2-5 may be understood in the same way.

(5) The notation – indicates that an upgraded system that is achieved through reduction technique cannot be equivalent to the upgraded system that is obtained through a duplication method.

8. Conclusions

The reliability and the performance of the Radar system is upgraded for the components with mixture distribution and delayed time. Four techniques are used to improve the reliability of the Radar model namely reduction, hot, cold, and imperfect duplication methods. The reliability function and mean time to failure for the original and upgraded systems are obtained. The Reliability equivalence factors and γ-fractiles are derived to differentiate between original and upgraded system. The simulation results indicated that the upgraded system is better than the original one. Furthermore, it is shown that the cold duplication is the best method for improving, but it takes too much time to give the results.

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